Will Society benefit most by finding the solutions to The Millennium Prize Problems or Hilbert's Problems?

It was a bright August morning in Paris when David Hilbert walked onto the stage to address a gathering of mathematicians in the Sorbonne. Theirs was a world on the cusp of change. Even as Hilbert stood before the hush of an expectant audience, around him Paris bustled with the excitement of the 1900 Summer Olympics, where women competed for the first time, and the first black athlete received a long-awaited medal, foreshadowing the overthrowing of old ways, a society that would soon be changed beyond all recognition. Also in Paris the Exposition Universelle was well underway, an international fair to celebrate scientific achievement and look forward to what the new century might bring. And indeed Hilbert spoke of the closing of a great epoch, an unknown future, his words resonant with anticipation of all that humankind might learn and achieve. There was a sense, surely, that this was a moment of some significance.

And Hilbert had chosen this moment to lay down a challenge. He presented a list of 23 unsolved problems in pure mathematics. They covered such diverse subjects as number theory and pure logic, geometry and topology – the study of shapes and surfaces – and even an attempt to 'axiomatize' physics, in essence to devise a grand 'theory of everything'. This, he hoped, would provide inspiration for a new generation of researchers, and shape the direction of mathematical research for a hundred years to come. Hilbert's problems were enthusiastically tackled, and the majority have now been solved. Even those that have not, have provided valuable results of their own – not least the realisation that some things cannot be known; not every question has an answer. But Hilbert's list received its fair share of criticism in the years that were to come, and to this day some are dubious about the merits of the particular problems he selected. The lack of 'applications' is a common observation. But this is surely to miss the point. When Hilbert spoke of 'the creative power of pure reason', he was aiming to inspire something more fundamental and radical than the utilitarian practicality of applied mathematics.

In 2000 the Clay Mathematics institute, inspired by Hilbert a hundred years earlier, released a new list of seven Millenium Prize Problems, including the now infamous Riemann hypothesis from Hilbert's list. This unsolved problem concerns the distribution of the prime numbers and if it were to be untrue, would have far-reaching effects on many accepted truths in mathematics and computer science. Added to the list is the perhaps equally notorious P=NP conjecture, relating to the computational time required to solve various classes of mathematical problems. The assumption that it is *not* true underpins the security of many of our computational systems; if it is ever proved true, we have a problem.

But if the average person was to come across the Hilbert and Millennium Prize problems, he or she is surely less likely to ponder which are of more value to society, than whether either are of any value at all. Or indeed of much interest. It is fashionable to deride mathematics as dull and irrelevant, particularly in its purer and more abstract forms.

Governments, understandably, prefer to direct limited funding towards research expected to generate business revenue and solve current health or social problems. Even proponents of 'blue sky' research tend to be thinking more of medical and technical fields than pure mathematics.

But the history of pure mathematics is full of great feats of human endeavour and heroism - consider Euler, formulating one theorem after the next, working on as illness and cataracts gradually took his sight. Or Sophie Germain, barred from formal study or professional recognition because of her gender, yet winning the esteem of the great mathematicians of her day, and making invaluable contributions to the field of number theory. What drove them perhaps was a love of mathematics for its own sake, a desire to know, to understand, a sense that in developing, defining, proving fundamental truths about the nature of numbers and logic, we would better understand our universe and all the things in it. Perhaps they considered 'applications'. But living long before the computer was dreamed of, before even the first 'difference engine' was a spark in the imagination of Charles Babbage, they could never have imagined that their results in number theory would form the basis of one of the most widely used cryptographic algorithms in the modern world, vital for data encryption and secure electronic commerce to this day. Real-world applications in mathematics, like many of the best things in life, are often found when we are least looking for them.

So which list will bring the greater benefit to society? It is far, far too soon to say. Pure mathematics does not lend itself well to short-term cost-benefit analysis, but as we have seen the benefits can be truly spectacular and startling. Ask the question again in a thousand years, and we may be in a position to judge.